Character Tables for $\mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$

representation φ $\rightsquigarrow$ $\chi_\varphi \in L^c(G)$
character

$\lambda^1, \ldots, \lambda^s$ complete list of irreducible reps of G
(pairwise inequivalent)

with characters $\chi_1, \ldots, \chi_s$

$\Rightarrow$ $\varphi \sim m_1 \lambda^1 \oplus \cdots \oplus m_s \lambda^s$ where

$$m_k = \langle \chi_k, \chi_\varphi \rangle$$

Example: $G = \mathbb{Z}_4 = \langle a \rangle$

	e	a	a^2	a^3
χ_1	1	1	1	1
χ_2	1	i	-1	$-i$
χ_3	1	-1	1	-1
χ_4	1	$-i$	-1	i

$$\langle \chi_2, \chi_2 \rangle = \frac{1}{4}\left[\chi_2(e)\overline{\chi_2(e)} + \chi_2(a)\overline{\chi_2(a)} + \chi_2(a^2)\overline{\chi_2(a^2)} + \chi_2(a^3)\overline{\chi_2(a^3)} \right]$$

$$= \frac{1}{4}\left[(1)(1) + (i)(-i) + (-1)(-1) + (-i)(i) \right]$$

$$= 1 \quad \checkmark$$

$$\psi : \mathbb{Z}_4 \longrightarrow GL_2(\mathbb{C})$$

$$\psi_{a^k} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^k$$

	e	a	a^2	a^3	$\langle \chi_\psi, \chi_k \rangle$
χ_ψ	2	0	-2	0	
χ_1	1	1	1	1	0
χ_2	1	i	-1	$-i$	1
χ_3	1	-1	1	-1	0
χ_4	1	$-i$	-1	i	1

Example:

$G = \mathbb{Z}_2 \times \mathbb{Z}_2 = \{e, a, b, c\}$

	e	a	b	c
χ_1	1	1	1	1
χ_2	1	1	-1	-1
χ_3	1	-1	1	-1
χ_4	1	-1	-1	1